Belief contraction through safe formulas

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Abstract[1]

In this paper we consider approaches to belief revision and contraction in modal logics that are based on a translation of an agent's beliefs in modal logic to first-order logic, where revision or contraction is performed before translating the result back into the original modal language. Contraction presents a unique problem, as theorems of the modal language can be accidentally removed, which would destroy the correspondence between a system of modal logic and its first-order simulation. We propose a solution for this problem based on defining safe-to-remove-parts of target formulas for contraction.

Introduction

Interest in the area of theory change started with a paper by Alchourrón, Gärdenfors, and Makinson [1]. The AGM theory of belief change explores questions arising when some agent receives new information that may be inconsistent with previously held beliefs, triggering the need to get rid of some old information in order to accommodate the new. The question is, what principles should govern how previous beliefs are changed when new information is learned? There are two options: *expansion* and *revision*. Expansion is simply the addition of new information to the existing belief set, but if the new information is inconsistent with the existing belief set, this can only lead to contradiction. Revision is a more complicated operation since it accommodates the case where new information is inconsistent with existing beliefs. This leads us to another operation, which is responsible for getting rid of unwanted beliefs: *contraction*.

The AGM theory was originally conceived in terms of classical propositional logic, so one line or research has been to extend the AGM theory to non-classical logics [5]. Our interest lies in the area of extending the AGM theory to modal languages [3].

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Gabbay, Rodrigues and Russo [8] proposed a belief revision approach for non-classical logics based on a translation of an agent's beliefs and semantics of non-classical logics to classical logic and performing belief revision in classical logic. But defining a contraction operator that is analogous to their revision operator revealed a problem, in that translated modal theorems can be removed during contraction in classical logic [15]. In this paper we consider this problem and propose a method to prevent modal theorems from being removed during contraction.

The paper is organized as follows. In Section 1 we give an overview of the AGM theory. In Section 2 we introduce the basic modal language and the notions of truth and validity in modal logic. In Section 3 we look at a belief revision approach for non-classical logics based on a translation to classical logic and consider a mechanism of translation of modal logic to classical first-order logic. In Section 4 we discuss a problem of accidental modal theorem removal during contraction in classical logic. In Section 5 approaches to non-prioritized belief change are presented. In Section 6 we propose a method for blocking accidental theorem removal by isolating the "safe" part of a target formula that can be contracted.

1. Belief revision

Let K be an agent's belief set, which is closed under the Tarskian consequence operator, $K = Cn(K)$, and ϕ be a target proposition [1]. The operation of expansion is defined in the following way: $K + \phi = Cn(K \cup \{\phi\})$. The operations of revision and contraction, however, are not defined uniquely, but are constrained by the AGM postulates [1]. The revision and contraction operators are understood to provide the minimal conditions for rationally revising and contracting information, respectively.

The basic AGM postulates for contraction ($\div$) are as follows:

- $K \div \phi$ is a belief set (closure);
- $K \div \phi \subseteq K$ (inclusion);

- If $\phi \notin Cn(K)$, then $K \subseteq K \div \phi$ (vacuity);
- If $\phi \notin Cn(\emptyset)$, then $\phi \notin Cn(K \div \phi)$ (success);
- $K \subseteq Cn\big((K \div \phi) \cup \{\phi\}\big)$(recovery);
- If $\phi \equiv \psi$, then $K \div \phi \equiv K \div \psi$ (extensionality).

The AGM postulates for revision $(*)$ are formulated as follows:

- $K * \phi$ is a belief set (closure);
- $\phi \in K * \phi$ (success);
- $K * \phi \subseteq Cn(K \cup \{\phi\})$ (inclusion);
- If $\neg\phi \notin K$, then $Cn(K \cup \{\phi\}) \subseteq K * \phi$ (vacuity);
- If $\neg\phi \notin Cn(\emptyset)$, then $K * \phi$ is consistent under Cn (consistency);
- If $\phi \equiv \psi$, then $K * \phi \equiv K * \psi$ (extensionality).

Contraction and revision can be defined through each other using the Levi or the Harper identities.

Harper identity: contraction by a belief can be obtained through revision by negation of this belief and then by elimination of what was not in the original belief set:

$$K \div \phi \;\equiv\; (K \;* \neg\phi) \cap K.$$

Levi identity: revision by a belief corresponds to first contraction by the negation of the target belief and then expanding by the target belief:

$$K * \phi \;\equiv\; (K \div \neg\phi) + \phi.$$

2. Modal logic

Let's introduce the basic modal language and define the notions of truth and validity in modal logic [2].

Languages of propositional modal logic are propositional languages to which modal operators of possibility ($\Diamond$, 'diamond') and necessity ($\Box$, 'box') have been added. The basic modal language is defined using a countable set of proposition variables Φ with elements $p, q, r, \ldots$, and a unary modal operator $\Diamond$. The well-formed formulas ϕ of the basic modal language are constructed using the following format:

$$\phi ::= p | \bot \,| \neg\phi | \phi \vee \psi | \,\Diamond\, \phi,$$

where p ranges over elements of Φ. The necessity operator is defined as the dual of the possibility operator ($\Box\phi \coloneqq \neg \Diamond \neg\phi$). The interpretation of our modal language can be done in two distinct ways: at the level of models and at the level of frames. Both levels (of models and frames) are significant in their own way.

A frame for the basic modal language is a pair $F = (W, R)$, such that W is a non-empty set of worlds (states), and R is a binary accessibility relation on W.

A model for the basic modal language is a pair $M = (F, V)$, where F is a frame for the basic modal language, and V is a valuation function assigning to each proposition letter $p \in \Phi$ a subset $V(p)$ of W.

Suppose w is a state in a model $M = (W, R, V)$. The notion of a formula ϕ being true at a state w in a model M is defined inductively as follows:

${}^{M}_{w}\Vdash p$ iff $w \in V(p)$, where $p \in \Phi$,

${}^{M}_{w}\Vdash \bot$ never,

${}^{M}_{w}\Vdash \neg\phi$ iff not ${}^{M}_{w}\Vdash \phi$,

${}^{M}_{w}\Vdash \phi \vee \psi$ iff ${}^{M}_{w}\Vdash \phi$ or ${}^{M}_{w}\Vdash \psi$,

${}^{M}_{w}\Vdash \Diamond\, \phi$ iff for some $v \in W$ with Rwv we have ${}^{M}_{v}\Vdash \phi$.

A formula ϕ is valid in a model M (${}^{M}\Vdash \phi$) if it is true at all states in M (${}^{M}_{w}\Vdash \phi$, for all $w \in W$).

A formula ϕ is true at a state w in a frame F (${}^{F}_{w}\Vdash \phi$) if ϕ is true at w in every model (F, V) based on F; ϕ is valid in a frame F (${}^{F}\Vdash \phi$) if it is true at every state in F (${}^{F}_{w}\Vdash \phi$, for all $w \in W$), ϕ is an axiom of modal logic.

The concept of validity in a frame interprets modal formulas by abstracting away from the influence of valuations. Modal axioms represent properties of an accessibility relation of a frame (transitivity, reflexivity, etc.).

3. Belief revision in Modal logics

Gabbay, Rodrigues and Russo proposed a belief revision approach for non-classical logics whose semantics can be axiomatized in first-order logic [8]. The main idea is to define a belief revision operation for such logics via a standard belief revision operation for classical logic. The key steps of the approach are:

- translate the mechanics of the object logic L into classical first-order logic;
- perform the revision in first-order logic;
- translate the result back to the object logic L.

Let Δ be a non-classical logic theory; ϕ an input formula; $*$ an AGM revision operator in classical logic; τ a translation function from L into classical logic; A_L a sound and complete axiomatisation of semantic features of L in classical logic; Acc a characterisation of acceptable L-theories in first-order logic. Following [8], the revision operator $*_L$ in the logic L is defined as follows:

$$\Delta *_L \phi \;= \{\beta | \Delta^\tau * (\phi^\tau \wedge A_L \wedge Acc) \vdash \beta^\tau\},$$

where Δ^τ and $\phi^\tau$are the classical logic translations of Δ and ϕ respectively.

Semantic properties of the object logic L are preserved during revision process and the revised theory can be translated back to the object logic due to addition of A_L to the input formula. For modal logic the notion of inconsistency is identical to the notion of inconsistency in classical logic. Hence, there is no need to take into account Acc and the revision operator in modal logic can be simplified as follows:

$$\Delta *_L \phi \;= \{\beta | \Delta^\tau * (\phi^\tau \wedge A_L) \vdash \beta^\tau\}.$$

Gabbay et al. [8] apply this technique to normal modal logic, Belnap's four-valued logic and Łukasiewicz's many-valued logic.

Translation of modal logic to classical logic

Let's look at the mechanism of translation of modal logic to classical first-order logic described in [14] and [2]. Consider a first-order language corresponding to a basic modal language over a model $M = (W, R, V)$ with a binary predicate letter R for the accessibility relation, and unary predicate letters P, Q, R ... matching proposition variables $p, q, r, \ldots$. Let variables $x, y, z, \ldots$ range over the worlds in W. In our work, we assume that W is finite, some Φ will be finite, too. The translation mechanism is different for translation of modal formulas that are true at the level of models and that are valid at the level of frames.

Level of models (standard translation). The translation of an arbitrary modal formula that is true at a particular state of a model can be obtained via the mechanism of standard translation. The standard translation $ST_x(\phi)$ of a modal formula ϕ that is true at a state w is a first-order formula with one free variable x corresponding to w, defined inductively:

$$ST_x(p) = Px;$$

$$ST_x(\bot) = x \neq x;$$

$$ST_x(\neg\phi) = \neg ST_x(\phi);$$

$$ST_x(\phi \vee \psi) = ST_x(\phi) \vee ST_x(\psi);$$

$$ST_x(\Diamond \phi) = \exists y(Rxy \wedge ST_y(\phi));$$

$$ST_x(\Box\phi) = \forall y(Rxy \rightarrow ST_y(\phi)).$$

A formula ϕ that is valid in a model will have the correspondent first-order formula $\forall x\, ST_x(\phi)$.

The standard translation is not surjective. First-order formulas can be obtained for all modal formulas by the standard translation. But not all first-order formulas are the standard translations of some modal formulas or equivalent to the standard translations of some modal formulas (Fig. 1).

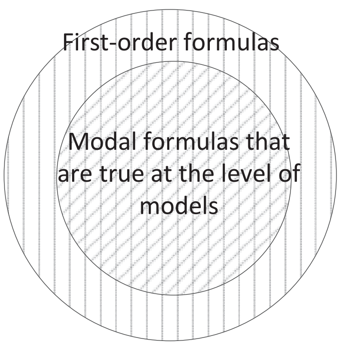

Fig. 1. Correspondence between modal logic and first-order logic on the level of models

Level of frames. It is not enough to quantify over all states of a frame to define validity in a frame, we also need to quantify over all possible valuations (consequently, over all subsets of a frame states). So, every modal formula that is valid at the level of frames corresponds to a second-order formula. This second-order formula may or may not have a first-order equivalent. Modal formulas can define frames that no first-order formula can. And there are first-order definable frame classes which no modal formula can define (Fig. 2). There is no unique way to define the translation between modal theorems and first-order formulas.

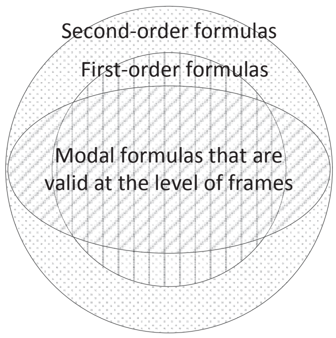

Fig. 2. Correspondence between modal logic and first-order logic on the level of frames

Examples of the correspondence between modal axioms and first-order formulas:

$\Box p \longrightarrow p$ corresponds to $\forall x Rxx$ (Reflexivity);

$\Box p \longrightarrow \Box\Box p$ corresponds to $\forall x \forall y \forall z(Rxy \wedge Ryz \longrightarrow Rxz)$ (Transitivity);

$p \longrightarrow \Box \Diamond p$ corresponds to $\forall x \forall y(Rxy \longrightarrow Ryx)$ (Symmetry).

Examples of modal axioms that correspond only to second-order formulas include McKinsey formula, $\Box \Diamond p \longrightarrow \Diamond \Box p$, Löb formula, $\Box(\Box p \rightarrow p) \rightarrow \Box p$ that defines frames with transitive R and well-founded R's converse.

4. Problem of "taboo" formulas contraction

In [15] the construction of contraction analogous to revision in [8] is considered. It is necessary to provide a protection mechanism to block modal theorem removal (there was no such problem in revision). The translation of an input formula, ϕ^τ, should be understood as (possibly) composed of two disjoint parts: contingent formulas, ${\phi^\tau}_{safe}$, which are "safe" to remove, and modal theorems, ${\phi^\tau}_{taboo}$, which should not be removed:

$$\phi^\tau = {\phi^\tau}_{safe} \wedge {\phi^\tau}_{taboo} .$$

The definition of AGM contraction, which is done in terms of the "safe" part of ϕ^τ, is

$$\Delta \div_L \phi = \left\{\beta \middle| \mathrm{Cn}(\Delta^\tau \cup A_L) \div {\phi^\tau}_{safe} \vdash \beta^\tau\right\}.$$

Example 1

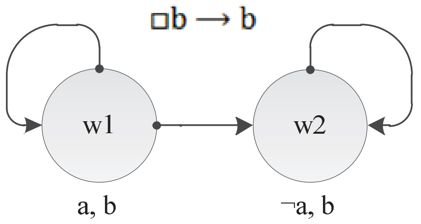

Fig. 3. Modal model *M*

Given a modal model $M = (W, R, V)$ with an axiom $\Box b \rightarrow b$ and an alphabet Φ (Fig. 3), where

$\Phi = \{a, b\}$,

$W = \{w_1, w_2\}$,

$R = \{(w_1, w_2), (w_1, w_1), (w_2, w_2)\}$,

$V(a) = \{w_1\}, V(b) = \{w_1, w_2\}$,

suppose it is necessary to contract $\phi = \Box b \rightarrow b$ from the world w_2.

The agent's belief set, Δ, includes implicit knowledge about accessibility relations between worlds in a model, values of each propositional variable in each state, and modal formulas that are true in worlds in a model.

Let's pick out $\Delta^\tau, \phi^\tau, A_L$. A predicate letter R corresponds to the accessibility relation R in M; and unary predicate letters A, B match the proposition variables a, b; variables w_1, w_2 correspond to the worlds w_1, w_2 of M.

$$\Delta^\tau = Cn(Rw_1w_2, Rw_1w_1, Rw_2w_2, \\ Aw_1, Bw_1, \neg Aw_2, Bw_2),$$

$$\phi^\tau = \forall y(Rw_2y \rightarrow By) \rightarrow Bw_2 \\ = \big((Rw_2w_1 \rightarrow Bw_1) \rightarrow Bw_2\big) \\ \wedge \big((Rw_2w_2 \rightarrow Bw_2) \rightarrow Bw_2\big) \\ = \big((Rw_2w_1 \wedge \neg Bw_1) \vee Bw_2\big) \\ \wedge \big((Rw_2w_2 \wedge \neg Bw_2) \vee Bw_2\big),$$

$A_L = \forall x Rxx = Rw_1w_1 \wedge Rw_2w_2$ – "taboo" formula.

To contract a disjunction $\phi \vee \psi$ we have to contract both ϕ and ψ $(A \div (\phi \vee \psi) \equiv (A \div \phi) \cap (A \div \psi))$. To contract a conjunction $\phi \wedge \psi$ we have to choose one of 3 options: to contact ϕ or ψ or both ϕ and ψ [1, 10].

Using these rules, we will get the following possible way to perform contraction in our example:

$$(\Delta^\tau \cup A_L) \div \phi^\tau \\ = (\Delta^\tau \cup A_L) \div \big((Rw_2w_1 \wedge \neg Bw_1) \vee Bw_2\big) \\ \wedge \big((Rw_2w_2 \wedge \neg Bw_2) \vee Bw_2\big) \\ = (\Delta^\tau \cup A_L) \div \big((Rw_2w_2 \wedge \neg Bw_2) \vee Bw_2\big) \\ = ((\Delta^\tau \cup A_L) \div (Rw_2w_2 \wedge \neg Bw_2)) \\ \cap ((\Delta^\tau \cup A_L) \div Bw_2) \\ = ((\Delta^\tau \cup A_L) \div Rw_2w_2) \cap ((\Delta^\tau \cup A_L) \div Bw_2).$$

One of ways to perform $(\Delta^\tau \cup A_L) \div \phi^\tau$ includes contraction by Rw_2w_2, as $A_L \vdash Rw_2w_2$, we will contract A_L as well. This leads to the loss of $\Box b \rightarrow b$ axiom in the resulted theory. The example shows that nothing stops us from removing modal axioms during contraction in classical logic and it's necessary to define a way to contract only by the "safe" part of the input formula.

Observation. There is a question of how to deal with contraction of the formulas coming from the translation of a valuation function, for example Bw_1, Bw_2. In a belief set in classical logic each proposition may be true, false or we may have no information about it. But in a belief set in modal logic we have to know whether the proposition is true or false in each world.

Belief change that allows the agent to not fully accept new information (for example, accept only the "safe" part) is called non-prioritized belief change.

5. Non-prioritized belief change

AGM postulates require full acceptance of new input information and elimination of the information that contradicts the input formula. But this approach is not suitable for realistic examples, because the agent may not consider all news as equally trustworthy. The input information may not be better than the information that has already been accepted.

In non-prioritized belief change the success postulate is not necessarily satisfied (because it requires full acceptance of the input information).

Non-prioritized belief change is thoroughly considered and classified in [4, 9, 5]. Two types of classification for revision are proposed: classification based on outcome and classification based on how to revise, each class is illustrated by examples. Two described methods are applicable for our problem of "taboo" formulas contraction: selective revision and shielded contraction.

Selective revision

Selective revision allows the agent to fully accept the input formula, partially accept it, or not accept the input formula at all. Selective revision includes two steps: a decision of what to accept and then, if the decision is to accept the input formula or some part of it, revision.

Selective revision. Let K be a belief set, $*$ an operation for revision for K, and f a function from L to L. The selective revision $\circ$, based on $*$ and f, is the operation such that, for all sentences ϕ

$$K \circ \phi = K * f(\phi),$$

where f is the transformation function on which $\circ$ is based. The transformation function determines if the input formula will be fully accepted, partially accepted or rejected.

Typically, the transformation function has property $\vdash \phi \rightarrow f(\phi)$.

Shielded and selective contraction

During non-prioritized shielded contraction some non-taulogical beliefs can be shielded from removal. The main idea is to divide the language into two parts, the retractable ("safe") and the unretractable ("taboo") sentences, and allow contraction only of the retractable sentences.

Shielded contraction [7]. Let K be a belief set, $\div$ an AGM contraction operator and R a subset of L (set of retractible sentences). Then $\div_o$ is the shielded AGM contraction induced by $\div$ and R if and only if:

$$K \div_o \phi = \begin{cases} K \div \phi, if\ \phi \in R \\ K, otherwise \end{cases}$$

The disadvantage of such contraction is that we cannot contract a part of the input sentence. So, it is reasonable to define contraction by analogy with selective revision that will be able to partially remove the input formula.

Selective contraction. Let K be a belief set, $\div$ an operation for contraction and f a transformation function from L to L. The selective contraction $\div_s$, based on $\div$ and f, is the operation such that for all sentences ϕ

$$K \div_s \phi = K \div f(\phi).$$

The transformation function defines if the input formula will be fully contracted, partially contracted or remain.

6. Transformation function

Let's define the transformation function for our problem of "taboo" formula removal. The transformation functions are a key mechanism for blocking accidental theorem removal. They should pick out only the "safe" part of the input formula ϕ^τ according to modal theorems A_L:

$$f(\phi^\tau, A_L) = \phi^\tau{}_{safe}.$$

If an arbitrary formula is "safe" to remove, it means that the formula is not a logical consequence of a set of formulas which include modal theorems. Therefore, modal theorems will not be removed together with the formula.

To check whether the target formula is a logical consequence of a set of formulas including modal theorems we can represent modal theorems and the target formula in full conjunctive normal form (FCNF) and look at the resulting conjuncts. All conjuncts in the target formula that appear among modal theorems conjuncts are prohibited from being contracted. The remaining conjuncts are "safe" to remove.

So, representing the input formula as FCNF allows us to pick out "safe" and "taboo" parts $\left(\phi^\tau = \phi^\tau{}_{safe} \wedge \phi^\tau{}_{taboo}\right)$. Reduce A_L, ϕ^τ to FCNF. Let ϕ^τ be a conjunction of $A_1, \dots, A_n$; A_L a conjunction of $T_1, \dots, T_m$. The "safe" part of ϕ^τ should not contain elements of $T_1, \dots, T_m$ and the transformation function f is defined in the following way:

$$\phi^\tau = A_1 \wedge \dots \wedge A_n,$$

$$A_L = T_1 \wedge \dots \wedge T_m,$$

$$f(\phi^\tau, A_L) = \phi^\tau{}_{safe} = \wedge\left(\{A_1, \dots, A_n\} \backslash \{T_1, \dots, T_m\}\right).$$

The "taboo" part of ϕ contains all the conjuncts that appear in ϕ^τ and in A_L and be defined as follows:

$$\phi^\tau{}_{taboo} = \wedge\left(\{A_1, \dots, A_n\} \cap \{T_1, \dots, T_m\}\right).$$

It is easy to see that the defined transformation function satisfies the property

$$\vdash \phi^\tau \to f(\phi^\tau, A_L).$$

The contraction operator described in [15] will have the following definition in terms of the transformation function:

$$\Delta \div_L \phi = \{\beta | (\Delta^\tau \cup A_L) \div f(\phi^\tau) \vdash \beta^\tau\}.$$

Example 1 (continued)

We need to get rid of predicates in A_L, ϕ^τ to be able to work with FCNF. Let's define new variables for each predicate with indexes corresponding to the states in the predicate (e.g. B_1 for Bw_1, P_{12} for Pw_1w_2). FCNF for A_L, ϕ^τ should be constructed from all variables appearing in A_L, ϕ^τ. Consider FCNF for ϕ^τ :

$$\begin{aligned}
\phi^\tau = & \left((Rw_2w_1 \wedge \neg Bw_1) \vee Bw_2\right) \\
& \wedge \left((Rw_2w_2 \wedge \neg Bw_2) \vee Bw_2\right) \\
& = \left((R_{21} \wedge \neg B_1) \vee B_2\right) \wedge \left((R_{22} \wedge \neg B_2) \vee B_2\right) \\
& = (R_{21} \vee B_2) \wedge (\neg B_1 \vee B_2) \wedge (R_{22} \vee B_2) \\
& = (R_{11} \vee R_{21} \vee R_{22} \vee B_1 \vee B_2) \\
& \wedge (R_{11} \vee R_{21} \vee \neg R_{22} \vee B_1 \vee B_2) \\
& \wedge (R_{11} \vee R_{21} \vee R_{22} \vee \neg B_1 \vee B_2) \\
& \wedge (R_{11} \vee R_{21} \vee \neg R_{22} \vee \neg B_1 \vee B_2) \\
& \wedge (R_{11} \vee \neg R_{21} \vee R_{22} \vee \neg B_1 \vee B_2) \\
& \wedge (R_{11} \vee \neg R_{21} \vee \neg R_{22} \vee \neg B_1 \vee B_2) \\
& \wedge (R_{11} \vee \neg R_{21} \vee R_{22} \vee B_1 \vee B_2) \\
& \wedge (\neg R_{11} \vee R_{21} \vee R_{22} \vee B_1 \vee B_2) \\
& \wedge (\neg R_{11} \vee R_{21} \vee \neg R_{22} \vee B_1 \vee B_2) \\
& \wedge (\neg R_{11} \vee R_{21} \vee R_{22} \vee \neg B_1 \vee B_2) \\
& \wedge (\neg R_{11} \vee R_{21} \vee \neg R_{22} \vee \neg B_1 \vee B_2) \\
& \wedge (\neg R_{11} \vee \neg R_{21} \vee R_{22} \vee \neg B_1 \vee B_2) \\
& \wedge (\neg R_{11} \vee \neg R_{21} \vee \neg R_{22} \vee \neg B_1 \vee B_2) \\
& \wedge (\neg R_{11} \vee \neg R_{21} \vee R_{22} \vee B_1 \vee B_2).
\end{aligned}$$

Since $A_L = Rw_1w_1 \wedge Rw_2w_2 = R_{11} \wedge R_{22}$, FCNF for A_L will consist of all possible conjuncts with R_{11} or R_{22} without negation. We exclude such conjuncts from ϕ^τ and obtain the "safe" part of the input formula:

$$\begin{aligned}
\phi^\tau{}_{safe} = & (\neg R_{11} \vee R_{21} \vee \neg R_{22} \vee B_1 \vee B_2) \\
& \wedge (\neg R_{11} \vee R_{21} \vee \neg R_{22} \vee \neg B_1 \vee B_2) \\
& \wedge (\neg R_{11} \vee \neg R_{21} \vee \neg R_{22} \vee \neg B_1 \vee B_2)
\end{aligned}$$

So, to prevent modal theorem removal we need to contract the agent's belief set by

$$\begin{aligned}
\phi^\tau{}_{safe} = & (\neg Rw_1w_1 \vee Rw_2w_1 \vee \neg Rw_2w_2 \vee Bw_2) \\
& \wedge (\neg Rw_1w_1 \vee \neg Rw_2w_1 \vee \neg Rw_2w_2 \vee \neg Bw_1 \\
& \vee Bw_2)
\end{aligned}$$

instead of

$$\begin{aligned}
\phi^\tau = & \left((Rw_2w_1 \wedge \neg Bw_1) \vee Bw_2\right) \\
& \wedge \left((Rw_2w_2 \wedge \neg Bw_2) \vee Bw_2\right).
\end{aligned}$$

Conclusions

Selective contraction with the defined transformation function allows us to contract only by the “safe” part of the input information and, consequently, preserve modal theorems in the resulting belief sets. Since Gabbay et al.’s approach is defined for all first-order axiomatized logics our results about the transformation function can be used for all these logics too.

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