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Is there a logic of information?

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Abstract: Information-based epistemology maintains that 'being informed' is an independent cognitive state that cannot be reduced to knowledge or to belief, and the modal logic *KTB* has been proposed as a model. But what distinguishes the *KTB* analysis of 'being informed', the *Brouwersche* schema (*B*), is precisely its downfall, for no logic of information should include (*B*) and, more generally, no epistemic logic should include (*B*), either.

Keywords: Modal logic, epistemic logic, *B* schema.

At the heart of Luciano Floridi's information-based epistemology is the claim that 'being informed' is a distinct cognitive state, one that is stronger than belief but different from knowledge (Floridi 2011). According to Floridi (2006), the box operator of the normal modal logic *KTB* captures the salient features of a subject S's state of being informed that *p*. Unlike mere belief, being informed is *factive*: S is informed that *p* only if *p* is true; in other words,

(*T*) $\Box p \rightarrow p$.

Unlike true belief, being informed is also *constructive*: S is informed that *p* only if S is informed that he is not informed that not-*p*. However, according to Floridi, S may well be informed that *p* without being informed that he is informed that *p*, which means that being informed is conceived to be neither transitive nor Euclidean, and therefore is both too weak for the modal system *S5* and independent of *S4*.

But what distinguishes the *KTB* analysis of being informed, the *Brouwersche* schema,

(*B*) $p \rightarrow \Box\Diamond p$,

is precisely its downfall, for no regular modal logic of information should include (*B*), and no regular epistemic logic should include it, either.

The problem is that *KTB* is trigger-happy. It licenses leaps to conclusions that no logic of information should abide by. To begin to see why, observe that *KTB* is closed under the inference rule RR,[1] by virtue of being a regular system (Chellas 1980: 235), and has as theorems a pair of known schemas that I shall call the 'Bombs Away LeMay' schemas, or BAM for short.

(*BAM 1*) $\Box(\Diamond p \rightarrow q) \rightarrow (p \rightarrow \Box q)$.

(*BAM 2*) $\Box(p \rightarrow \Box q) \rightarrow (\Diamond p \rightarrow q)$.

Consider a pair of examples which exercise each schema.

> **Case 1**. Everyone in Babylonia believes that the circumference of a circle is thrice its diameter plus one-eighth, and nobody believes that Hesperus is Phospherus. Now suppose that Hammurabi is informed of the following dilemma ($\Box$): either he is informed that the circumference of a circle is a rational number ($\Box p$) or Hesperus is Phosphorus (q). Through *BAM 1*, it follows that Hammurabi has the information that Hesperus is Phosphorus ($\Box q$) if the circumference of a circle is not a rational number ($\sim p$).[2] So, while Hammurabi does not believe that Hesperus is Phosphorus, he nevertheless has the information that Hesperus is Phosphorus.

Case 1 illustrates that information states are not belief states. The mere fact that Hesperus is Phosphorus suffices for Hammurabi to have the information that either he is informed that the circumference of a circle is a rational number or Hesperus is Phosphorus, even though he falsely believes that the circumference of a circle is rational and disbelieves that Hesperus is Phosphorus.

> **Case 2**. Smokey is informed that ($\Box$): there is an alarm (p) only if Smokey is informed that there is a fire ($\Box q$). Courtesy of *BAM 2*, if Smokey is not informed that there is no alarm ($\Diamond p$), then there is a fire (q).

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[1] Rule RR states that if $(p \,\&\, q) \rightarrow r$ is a theorem, then $(\Box p \,\&\, \Box q) \rightarrow \Box r$ is too.

[2] Specifically, *BAM 1* here is $\Box(\Diamond{\sim}p \rightarrow q) \rightarrow ({\sim}p \rightarrow \Box q)$ and p codes 'the circumference of a circle is a rational number'.

Ignorance is bliss. Provided that Smokey is informed either that there is no alarm or that he is informed that there is a fire, BAM 2 licenses Smokey to deduce that there is a fire if it is merely consistent with his information that there is an alarm.

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One might wonder how to avoid BAM but a look at the proofs below reveals the (B) schema to be a deep source of trouble for any system that includes (K). For not only is BAM set off in every normal system that includes (B), BAM is set off in every regular (non-normal) system that includes (B), too.[3]

BAM proofs

	BAM 1			*BAM 2*	
1)	$\Box(\Diamond p \to q) \to (\Box\Diamond p \to \Box q)$	(K)	1)	$(\Box{\sim}\Box q \,\&\, \Diamond p) \to \Diamond({\sim}\Box q \,\&\, p)$	($K\Diamond$)
2)	$\Box(\Diamond p \to q) \to ({\sim}\Box q \to \Diamond\Box{\sim}p)$	Rewrite 1	2)	$\Box(p \to \Box q) \to (\Diamond p \to \Diamond\Box q)$	Rewrite 1
3)	$\Diamond\Box{\sim}p \to {\sim}p$	($B\Diamond$)	3)	$\Diamond\Box q \to q$	($B\Diamond$)
4)	$\Box(\Diamond p \to q) \to ({\sim}\Box q \to {\sim}p)$	2,3 *PL*	4)	$\Box(p \to \Box q) \to (\Diamond p \to q)$	2, 3 *PL*
5)	$\Box(\Diamond p \to q) \to (p \to \Box q)$	Rewrite 4			

Because the BAM schemas appear in the regular systems *EMCB* through *S5*, this class of BAM systems pose a direct challenge to the claim that 'being informed' is a distinct epistemic modal. For unlike the problem posed by information closure (Floridi 2011, Dretske 2006), which is one that every regular system must face, the defining controversy for Floridi's logic of information is its doomed commitment to both *closure* and *symmetry*.

The inclusion of *S5* among the BAM systems also throws new light on the problem of interpreting the *S5* box operator as an epistemic modality. Since Hintikka's study of epistemic logic (1962), the received view has been that *S5* is ruined by the unwarranted powers of *negative introspection* afforded by the (*5*)

[3] Compare to Allo (2011), who proposes a weakening of the *KTB* analysis that nevertheless succumbs to BAM.

schema.[4] Add Williamson's brief against the *positive introspection* condition encoded by the (*4*) schema (Williamson 2000), and it is only natural to consider *KTB* as an alternative to both *S4* and *S5*. Nevertheless, *KTB* is a nonstarter for epistemic modalities and its ineligibility stems from the omission of the (*5*) schema. Contraposed, the (*5*) schema says that you are informed that *p* is false if the falsity of *p* is not *luminous* to you, which is to say, if you are not informed that you are not informed that *p* is false. While strong, perhaps unreasonably so, (*5*) nevertheless specifies a condition of a single type. The (*B*) schema by contrast mixes form. For the (*B*) schema says that the falsity of *p* is not luminous to you only if *p* is false. In other words, (*B*) ties the standing of a cognitive state, the failure of luminosity, to the standing of the bare semantic fact of whether *p* holds. The problem is that *KTB* is left without the means to fully explain how a semantic denotation determines a cognitive state. That connection is left a mystery without (*5*). So, exchanging *S5* for *KTB* amounts to trading an implausible explanation provided by (5) for no explanation at all.

Because the (*5*) schema merely puts forward the best face possible for the (*B*) schema, negative introspection is better viewed as a byproduct of a collision within *S5* involving three more fundamental normative principles: the *factivity* condition afforded by the (*T*) schema, the *transitivity* condition afforded by the (*4*) schema,[5] and the *symmetry* condition afforded by the (*B*) schema.

The fact that (*B*) is a bigger problem for epistemic logics than (*5*) muddies the water for epistemic logics built atop *S5*, such as dynamic epistemic logic (van Ditmarsch, van der Hoek, and Kooi 2007, van Bentham 2011). The rationale for viewing *S5* as a suitable vehicle for the dynamic turn in modal logic, for instance, is faith in the bounty of weaker systems that also support the dynamic extensions on offer. If *S5* is too strong, the reasoning goes, surely one of the many possible weaker systems will pass muster. But the (B) schema's reliance upon (5) for a plausible reading of the epistemic modality suggests that this promise of abundance is without providence. It may simply be that *S5* is the least-worst choice of the lot.

[4] Schema (*5*) is $\Diamond p \rightarrow \Box \Diamond p$.

[5] Schema (4) is $\Box p \rightarrow \Box \Box p$.

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There is an important difference between the relatively benign role that (B) plays in provability logics and the calamitous one it plays within epistemic logics. Referring to (B) as the Brouwersche schema within the context of provability logic is transparently sensible, for the noun 'constructability' is procedural but non-cognitive. The bad behavior of (B) within regular epistemic logics, however, is neither transparent nor sensible. To mark this hazard, and to preserve Brouwer's good name, I propose that epistemic logicians hereafter refer to (B) as the *Blog schema*.

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So, is there a logic of information? Not within the BAM systems. Outside the BAM systems, however, there are few options available to ground information-epistemology whilst maintaining that 'being informed' is distinct from both knowledge and belief. The insight from the logic of information is not to do with novel epistemic states or a newfangled epistemology. Rather, the lesson to draw is a cautionary one for all of epistemic logic to heed: symmetry and closure do not combine.[6]

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