On the Structure of Rational Acceptance: Comments on Hawthorne and Bovens

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Abstract.
The structural view of rational acceptance is a commitment to developing a logical calculus to express rationally accepted propositions sufficient to represent valid argument forms constructed from rationally accepted formulas. This essay argues for this project by observing that a satisfactory solution to the lottery paradox and the paradox of the preface calls for a theory that both (*i.*) offers the facilities to represent accepting less than certain propositions within an interpreted artificial language and (*ii*) provides a logical calculus of rationally accepted formulas that preserves rational acceptance under consequence. The essay explores the merit and scope of the structural view by observing that some limitations to a recent framework advanced James Hawthorne and Luc Bovens are traced to their framework satisfying the first of these two conditions but not the second.

Keywords: Lottery paradox; paradox of the preface; epistemic closure; probability, probabilistic logic, rational acceptance.

1.

The *lottery paradox* (Kyburg 1961) arises from considering a fair 1000 ticket lottery that has exactly one winning ticket. If this much is known about the execution of the lottery it is therefore rational to accept that one ticket will win. Suppose that an event is very likely if the probability of its occurring is greater than 0.99. On these grounds it is rational to accept the proposition that ticket 1 of the lottery will not win. Since the lottery is fair, it is rational to accept that ticket 2 won't win either—indeed, it is rational to accept for any individual ticket i of the lottery that ticket i will not win. However, accepting that ticket 1 won't win, accepting that ticket 2 won't win, ..., and accepting that ticket 1000 won't win entails that it is rational to accept that no ticket will win, which entails that it is rational to accept the contradictory proposition that one ticket will win and no ticket will win.

The *paradox of the preface* (Makinson 1965) arises from considering an earnest and careful author who writes a preface for a book he has just completed. For each page of the book, the author believes that it is without error. Yet in writing the preface the author believes that there is surely a mistake in the book, somewhere, so offers an apology to his readers. Hence, the author appears to be committed to both the claim that every page of his book is without error and the claim that at least one page contains an error.

Abstracted from their particulars, the lottery paradox and the paradox of the preface are each designed to demonstrate that three attractive principles for governing rational acceptance lead to contradiction, namely that

1. It is rational to accept a proposition that is very likely true,
2. It is not rational to accept a proposition that you are aware is inconsistent, and
3. If it is rational to accept a proposition A and it is rational to accept another proposition A', then it is rational to accept $A \wedge A'$

are jointly inconsistent. For this reason, these two paradoxes are sometimes referred to as *the paradoxes of rational acceptance*.

These paradoxes are interesting because of the apparent price exacted for giving up any of the three principles governing rational acceptance. Abandoning the first principle by restricting rational acceptance to only certainly true propositions severely restricts the range of topics to which we may apply logic to draw "sound" conclusions, thereby threatening to exclude the class of strongly supported but possibly false claims from use as non-vacuous premises in formally represented arguments. Giving up the last principle by abandoning logical closure operations for accepted propositions clouds our understanding of the logical form of arguments whose premises are rationally accepted but perhaps false, thereby threatening our ability to distinguish good argument forms from bad. Finally, adopting a strategy that denies the second principle offers little advantage on its own, since even a paraconsistent approach that offers a consequence operation that does not trivialize when applied to a set containing a contradictory proposition must still specify a closure operation for rationally accepted propositions that reconciles the general conflict between the first and third legislative principles.

When considering a strategy to resolve a paradox it is worth remarking that simply avoiding inconsistency is not necessarily sufficient to yield a satisfactory solution since consistency may be achieved merely by dropping one of the conditions necessary to generate the antinomy. Besides restoring consistency, a satisfactory resolution must also address the motivations behind the principles that generate the paradox in the first place. There are two ways one can do this. The first type of response is to reject one or more of the principles and then explain how to get along without principles of this kind. The thrust of this approach is to claim that a purported paradox is really no paradox at all but rather a mistake arising from a commitment to a dubious principle. The second type of reply regards the constituent principles of a paradox as all well-motivated, if ill-formulated, so regards the paradox as genuine. The aim of this type of reply is to offer a substantive solution to the paradox, which is a solution that revises one or more of the original principles so that they consistently capture the key features that motivated adopting the original principles.

In the case of the lottery paradox and the paradox of the preface a solution of the second type is required. Namely, a satisfactory solution to these paradoxes should provide a sufficiently expressive language for representing accepting less than certain propositions and also provide a sufficiently powerful logic to model entailments made in cogent arguments involving uncertain but rationally accepted premises.[1]

2.

This description of the paradoxes of rational acceptance and what should be expected from a solution is fairly standard. However, in this essay I propose refining the standard view by adding a requirement that every proposed solution should satisfy. The requirement concerns minimal syntactic capabilities that a formal system's language should possess. More specifically, the proposal is to require that a system's formal language be expressive enough to construct compound rationally accepted formulas. This language requirement may be thought of as a *structural constraint* on the formal system underlying any proposed solution to the paradoxes. For this reason, I refer to this proposal as the *structural view of rational acceptance*.

The structural view is motivated by observing that the problem raised by the paradoxes of rational acceptance is a general one of how to reconcile the first and third legislative principles. But to study the general relationship between rational acceptance and logical consequence, we need to understand valid *forms* of arguments whose premises are rationally accepted propositions. This point suggests three conditions for us to observe. First, it is important to define the notion of rational acceptance independently of any particular interpreted structure, since this notion is serving as a semantic property that is thought to be preserved (in a restricted sense) under entailment. Second, to formally represent an argument composed of rationally accepted propositions we must have facilities for formally representing their combination within an object language. Finally, of formal languages that satisfy the first two properties, preference should be given to those within systems that make the relationship between rational acceptance and logical consequence transparent.

Notice that these conditions correspond to general properties that well-designed logical calculi enjoy. However, the most familiar logical calculus—the propositional calculus defined on the primitive Boolean connectives $\neg$ and $\vee$—is precisely the calculus that generates the paradoxes of rational acceptance. The structural view of rational acceptance then sees the problem raised by the paradoxes to be one of selecting the right calculus for rationally accepted formulas.

It is worth mentioning that the structural constraints are not jointly sufficient conditions for resolving the paradoxes since there are several ways a formal language could meet the first two constraints, and the notion of transparency that figures in the third condition is imprecise. No doubt other concerns will need to be brought to bear to select the correct class of logics for rational acceptance. However, the point of this essay is to argue that a logic of rational accepted formulas should at least satisfy these conditions. Viewing these paradoxes within frameworks that attempt to satisfy these minimal constraints will allow us to focus more precisely on the key open questions surrounding rational acceptance.

It is also important to note that there isn't anything necessarily mistaken about an unstructured logic. For instance, the operator Γ, defined over a set of accepted sentences X such that a proposition A is in the image set of $\Gamma(X)$ if and only if $X \cup \{\neg A\}$ is inconsistent, is an unstructured operator; yet, Γ is also sound. If Γ and $\neg$ were the only operators a logic featured, that logic also would be unstructured and sound. The problem with an unstructured consequence operator like Γ is that if we have a question whose answer turns on the syntactic details of how the manipulation of elements in X affects the appearance of A in the image set of $\Gamma(X)$, then Γ is the wrong theoretical tool to expect an answer to that question.

Thus the difference between structured and unstructured systems is not logical in the sense that each type of framework necessarily identifies different classes of rationally accepted propositions. Rather, the fundamental disagreement rests in what analytical resources are necessary to study arguments composed of rationally accepted propositions. The structural view holds that it is necessary for the object language to include connectives in order to express compound rationally accepted formulas and to define restricted logical consequence for rationally accepted formulas in terms of these connectives. An unstructured view does not.

Finally, note that the structural view has an important methodological consequence for rational acceptance studies. For if one accepts that what is needed is a formal language for rational accepted formulas, then research should move away from purely semantic approaches and toward the study of probabilistic logical calculi.

3.

To motivate the structural view it will be useful to consider an important framework that does not satisfy the structural constraints just discussed, the *logic of belief* developed by James Hawthorne and Luc Bovens in (Hawthorne and Bovens 1999). The point behind criticizing this particular framework is to show that certain limitations of that theory's solution to the paradoxes is traced to the unstructured logic underpinning the account. The reason that Hawthorne and Bovens's system is an excellent one for making my general point is that their system is very well developed: it is doubtful that their theory can be improved without adopting the structural constraints that are the focus of this essay.

Hawthorne and Bovens view the lottery paradox and the paradox of the preface to be problems involving how to identify rational beliefs resulting from composing probabilistic events (e.g., how many of the tickets I each judge as losing tickets may I conjoin and still rationally regard as losing tickets?) and how to identify rational beliefs resulting from the decomposition of compound probabilistic events (e.g., how short can my book be before my apology for mistakes in the preface becomes incoherent?). They frame their discussion of the paradoxes in terms of belief states for ideal agents who satisfy certain rational coherence constraints.

Hawthorne and Bovens's approach follows a proposal made by Richard Foley (Foley 1992) about how to construe the first legislative principle for rational acceptance. In Foley's paper he advances the *Lockean thesis*, which states that rational acceptance should be viewed as rational belief and that a rational belief is just a rational degree of confidence above some threshold level that an agent deems sufficient for belief. We'll say more about this principle shortly. Hawthorne and Bovens's project is to use the rationality constraints that come with probabilistic models of doxastic states—and which are built into the Lockean thesis by virtue of framing rational acceptance in terms of rational belief—in order to establish a correspondence between quantitative degrees of confidence and a qualitative notion of full belief. This correspondence then allows them to reconstruct a probabilistic model of belief for an agent given only that he is in a suitable context in which he has full beliefs that satisfy the Lockean thesis, and *vice versa*. The Hawthorne and Bovens proposal, then, is that from the constraints imposed by the Lockean thesis, an ideal agent's report of his (full) beliefs provides us with rational lottery-states and rational preface-states, which in effect yields a solution to the paradoxes since these states will include rational beliefs that are combinations of individual rational beliefs and rational beliefs that are detached from compound rational beliefs.

Hawthorne and Bovens regard this approach as a powerful framework for resolving the paradoxes of rational acceptance, stating that there is "a precise relationship between ... qualitative and quantitative doxastic notions" that "may be exploited to provide a completely satisfactory treatment of the *preface* and the *lottery*" paradoxes (Hawthorne and Bovens 1999, 244). The gist of their proposal is that representing the lottery paradox and the preface paradox in terms of the logic of belief yields enough insight into the relationship between qualitative and quantitative notions of rational acceptance to provide "a foundation for a very plausible account of the logic of rationally coherent belief" (Hawthorne and Bovens 1999, 244).

After summarizing their proposal in the next section, I will advance reasons for resisting both of these claims in Sections 5 and 6. Specifically, I will argue that the relationship between qualitative and quantitative notions of belief does not afford us results sufficient to construct a satisfactory solution to these paradoxes but, on the contrary, introduces obstacles to constructing such an account. Furthermore, I suggest that there is reason to doubt that Hawthorne and Bovens's framework provides a suitable foundation for the logic of rationally coherent belief.

4.

Hawthorne and Bovens's logic of belief is a theory that yields consistency constraints for an ideally rational agent α who grasps all logical truths. Two types of doxastic states for α are considered, a quantitative doxastic notion, called a *degree of confidence function*, and the qualitative notions of full belief and its complement.[2] The degree of confidence function, defined over a countable set $\mathcal{F}$ of propositions, is isomorphic to the classical probability measure, whereas belief and its complement are defined relative to a threshold point in the unit interval. The relationship between these two notions is given by the

Lockean thesis: α is said to believe a proposition A in $\mathcal{F}$ if and only if α's degree of confidence measure of A is greater than or equal to a threshold value q in the closed unit interval [0,1]. It is from this equivalence relation and the logical omniscience assumption for α that Hawthorne and Bovens derive the central results underpinning their proposal.

Their idea is to consider descriptions of α entertaining beliefs sufficient to generate an instance of the preface paradox and also of α entertaining beliefs sufficient to generate an instance of the lottery paradox, yielding belief states that are called *preface states* and *lottery states*, respectively. Within a sub-class of belief states the consistency constraints imposed by the degree of confidence measure allow one to derive a precise estimate of α's threshold value q, in cases where q is unknown. This is achieved by using α as an oracle to determine whether a proposed belief state (in an appropriately constrained context) satisfies both the Lockean thesis and the consistency constraints imposed by the degree of confidence measure over $\mathcal{F}$. The idea is that an ideally rational agent satisfying the Lockean thesis may be used as a semantic reference for determining the class of full beliefs, from which a quantitative probability model may be constructed. One may pass in the other direction as well—from a quantitative probabilistic doxastic notion to a qualitative model of the ideal agent's set of full beliefs—so long as the agent satisfies the Lockean thesis.

In the case of the preface, suppose there is a particular book with n pages and an agent α who believes that each page in this book is without an error yet also believes that there is an error on at least one of the pages. Hawthorne and Bovens refer to this as an *n-page preface state.* If q is a threshold value for belief, then α can consistently be in an n-page preface state only if $n \geq \frac{q}{1-q}$. Hawthorne and Bovens propose exploiting this inequality to fix a least upper bound on q when its value is unknown by solving the inequality for q rather than n, that is $\frac{n}{n+1} \geq q$. The idea is that if one doesn't know the value of an agent's threshold point for acceptance, q, one can provide a least upper bound for this value by placing α in preface states of varying size n and record the least value n where α satisfies the rationality constraints of the Lockean thesis.

In the case of the lottery, matters are slightly more complicated. Whereas placing α in various sized preface states is designed to fix the least upper bound on α's threshold value q, an analogous method for placing α in a restricted class of lottery states is intended to fix the greatest lower bound on q. The restricted class of lottery states, called *weak lottery contexts*, are just those that have at most one winning ticket—that is, for all $W_{i,j} \in \mathcal{F}$, α is certain that $\neg(W_i \wedge W_j)$ where $i \neq$

j. Adopting the phrase 'deems it possible that W' to express that α does not believe $\neg W$, Hawthorne and Bovens define an *m-ticket optimistic state* in a weak lottery context to be one in which an agent deems it possible that each m tickets of a lottery may win but that at most one ticket will win. When the threshold point q is defined, the agent may be in an m-ticket optimistic state only if $m < \frac{1}{1-q}$. When q is unknown, the greatest lower bound may be calculated by solving for q rather than n, that is $\frac{m-1}{m} < q$. The idea here is that if one doesn't know the value of an agent's threshold point for acceptance, then place α in various sized optimistic states and record the greatest value m where α satisfies the rationality constraints of the Lockean thesis.

The idea then is to combine these two results to fix an upper and lower bound on α's *quantitative* threshold of belief, q, by determining what α *qualitatively* believes in preface states that satisfy the weak lottery context restriction for optimistic states—namely, those contexts in which not more than one ticket wins and the set $\mathcal{F}$ of tickets (propositions) is finite—while satisfying the Lockean thesis. Suppose that α is in a context for belief that is an n-page preface state and also an n-optimistic state such that α believes he will not win with only $n-1$ tickets but deems it genuinely possible that he may win with n tickets. Hawthorne and Bovens's first result then is that α's threshold value for belief is some q such that $\frac{(n-1)}{n} < q \leq \frac{n}{n+1}$.

This estimate for q may be improved if additional restrictions are introduced. For instance, Hawthorne and Bovens introduce the notion of a *strong equiplausible lottery context*, which holds when α is certain that exactly one ticket wins and that the outcomes are equiprobable. Then a more precise estimate for q may be derived.

> In general, if in a *strong equiplausible lottery context* for an n ticket lottery an agent believes she will not win with only $m-1$ tickets [for $m \leq n$], but deems it *genuinely possible* that she may win with m tickets, then the agent's threshold value for belief is some number q such that $1-(\frac{m}{n}) < q \leq 1-\frac{m-1}{n}$ (Hawthorne and Bovens 1999, 254).

Their claim then is that the two kinds of propositional attitudes, qualitative belief and quantitative degree of confidence, show that ideal agents that satisfy the Lockean thesis may rationally entertain lottery beliefs and preface beliefs without contradiction.

> If *preface* and *lottery* beliefs are re-described in quantitative doxastic terms, their paradoxical features evaporate. In the *lottery* we realize that the likelihood that any given ticket will win is extremely low, yet this in no way contradicts our certainty that

> some ticket will win. In the *preface* we judge that the likelihood that any given page still contains an error is extremely low, yet this is perfectly consistent with our high degree of confidence that at least one error has been missed in a lengthy book (Hawthorne and Bovens 1999, 243).

However, there is reason to resist the claim that each paradox evaporates, if by 'evaporate' it is intended that the proposal provides a satisfactory solution to the preface and lottery paradoxes. For, as we've observed, one may dissolve these paradoxes by denying any one of the three legislative principles with which we began. In Hawthorne and Bovens's proposal the Lockean thesis satisfies the first principle, while the second is satisfied by consequence of adopting the classical probability measure that underpins modeling α as an agent who satisfies the Lockean thesis, irrespective of whether α's doxastic states are determined by a quantitative degree of confidence function or qualitative full belief. Hence, it is the third legislative principle that is rejected. Hawthorne and Bovens's strategy is to extract closure conditions for *particular* collections of beliefs from the semantics of the theory. The question remaining is whether this strategy resolves the general conflict between the first and third legislative principles.

5.

Hawthorne and Bovens proposal is built on an important insight, namely that a rationally acceptable n-element conjunction must itself be above threshold for acceptance rather than just assuring that each of the n conjuncts is above threshold. Thus, a closure condition for sets of rationally accepted propositions must account for the possible depletion of probability mass of conjoined probabilistic events. We might then think that the logic of belief does provide an account that generates structural rules. Such an account would propose using α to determine conjunctions of propositions, if any, that are above threshold when each conjunct is—that is, when $\mathrm{Pr}_\alpha(\bigwedge_{1\leq i\leq n} A_i) \geq q$ where $\{A_i : A_i \in \mathcal{F} \wedge \mathrm{Pr}_\alpha(A_i) \geq q\}$. The proposal would be to accept only those conjunctions that α does. Notice, however, that this isn't a structured closure operation since we do not have facilities within the object language for combining or decomposing accepted formulas to yield accepted formulas. Instead, what the theory provides is a description of a decision procedure in the metalanguage built around a semantic reference, α, who delivers a Yes or No reply to whether a candidate belief is rational to accept.

To illustrate this point, consider two rules that Hawthorne and Bovens discuss, labeled here as HB1 and HB2:

HB1. For all $n < \frac{q}{1-q}$, if α believes $\neg E_1, \alpha$ believes $\neg E_2, \ldots, \alpha$ believes $\neg E_n$, then α does not believe $(E_1 \vee E_2 \vee \ldots \vee E_n)$ (Hawthorne and Bovens 1999, p. 246).

HB2. For all $m \geq \frac{1}{1-q}$ and each $i \neq j$, if α is certain that $\neg(W_i \wedge W_j)$, and if α does not believe $\neg W_1, \alpha$ does not believe $\neg W_2, \ldots, \alpha$ does not believe $\neg W_{m-1}$, then for each $k \geq m$, α believes $\neg W_k$ (Hawthorne and Bovens 1999, p. 251).

Notice that HB1 and HB2, although sound, are unstructured. The point to notice is that HB1 and HB2 do not operate upon formulas but rather on states of belief. A consequence of this observation of particular importance is that there are no logical operators within Hawthorne and Bovens's logic of belief corresponding to the coordinating conjunctions appearing on the left-hand side of each rule. Hence, HB1 and HB2 are essentially meta-linguistic descriptions of decision procedures rather than inference rule schemata, since there are no formulas within Hawthorne and Bovens's framework to stand in as substitution instances for either rule. A consequence of this is that one cannot construct a proof within the logic of belief since there are no formulas from which to construct one. HB1 and HB2 are thus not logical rules of inference.[3]

This omission marks an important limitation to unstructured accounts. For it is clear that there is a logical distinction between a conjunction (disjunction) of rationally accepted propositions and a rationally accepted conjunction (disjunction); indeed, the paradoxes of rational acceptance are examples of arguments that invite us to ignore this distinction. But to formally evaluate arguments, the structure of formulas should reflect their meaning in a manner that clearly demonstrates the difference between conjunctions and disjunctions of propositions versus conjunctions and disjunctions of rationally accepted propositions. As we observed, Hawthorne and Bovens's logic of belief does not provide these resources.

To summarize, the first conclusion to draw about Hawthorne and Bovens's logic of belief is that it is not a logical calculus but instead is a specification for decision procedures that work by determining whether a belief state satisfies the semantic constraints of the theory, precisely as the unstructured operator Γ behaves with respect to propositions.

6.

Hawthorne and Bovens's analysis of the paradoxes of rational acceptance holds that the Lockean conception of belief, based on a probabilistic semantics with an acceptance level, offers a suitable foundation for resolving the paradoxes because it explains the relationship between qualitative belief, quantitative belief, and a quantitative threshold level for rational acceptance. In considering whether Hawthorne and Bovens's proposal provides a suitable foundation for a logic of rationally coherent belief we'll need to discuss the role that *qualitative belief* plays in the theory.

The first point to observe is that α's qualitative notion of belief does not play an essential role in the logic. By this I mean that their core theory starts with a known value for q and then classifies propositions by virtue of their probability measure with respect to q; full belief and its complement thus serve as derived notions. The criticism that Hawthorne and Bovens's account is unstructured applies to this core theory. The addition of qualitative belief to the logic plays no constructive part in resolving the issues raised in the previous sections, which is to say that the addition of qualitative belief does not in itself provide a means to construct rationally accepted formulas.

The main role that qualitative belief plays in the account is to provide an estimate of q when q is unknown, provided the agent satisfies the Lockean thesis—which, recall, includes the rationality constraints for probabilistic doxastic states—and an important restriction that I will return to shortly. It is precisely these rationality constraints that are built-in to the Lockean thesis that allows the theory to pass back and forth between qualitative and quantitative notions of belief. The main point to note here is that this estimation problem of the threshold parameter q is distinct from the issues stemming from not having a structured closure operation for sets of rationally accepted formulas.

Before pressing on, a remark on the restrictions necessary for full qualitative belief to estimate a value for q. By building their account around rational belief states, Hawthorne and Bovens need to restrict the scope of their theory to agents who are working with a qualitative notion of belief in order to pass from this notion to a quantitative estimate for q. They do this by specifying the kind of belief states in which the theory operates, generating a particular model of rationally accepted beliefs for that particular collection of beliefs. Hawthorne and Bovens propose approximating q from full qualitative belief within what they call weak lottery contexts—that is, in cases where we know there is no more than one mistakenly-accepted (believed and false) proposition out of a set of otherwise correctly accepted (believed and true) propositions. Note that rule HB2 incorporates the weak lottery context restriction.

However, in many cases involving rational acceptance the conditions for weak lottery contexts are not satisfied. We mustn't be tricked into thinking that an accidental feature of the lottery paradox thought experiment—such as that it is known for certain that no more than one ticket wins—picks out essential features that a logic for rational acceptance may always rely upon. In very many cases involving rationally accepted propositions errors of acceptance are independent (Kyburg 1997). For instance, in most government sponsored lottery drawings we do not know that there will be at most one ticket that will win, nor do we know that at most one plane will crash in a given year, nor do we typically know that no more than one sample will be biased among a collection of measurements. But the assumption that there is at most one mistakenly accepted proposition is necessary to define an m-element optimistic state, which in turn is used to approximate the greatest lower bound on q given α's belief states.

Admittedly, there is a degree of idealization we accept in modeling rational acceptance as belief states. What is important to notice here is that the restrictions used to approximate q from α's full belief states are more demanding than logical omniscience and should not pass without a note accounting for their cost.

These remarks also apply to Hawthorne and Bovens's proposal to reformulate their logic of belief in terms of *qualitative probability* (Hawthorne and Bovens 1999, Appendix 2). Considering why qualitative probability does not offer an improvement on their position with respect to providing a structured closure operation will put us in position to advance a reason to doubt their claim that the theory provides a good foundation for a logic of rational belief.

Qualitative probability theory stems from an observation of Frank Ramsey's (Ramsey 1931) that beliefs of ideally rational agents form a total order: it is the violation of this condition that underpins his Dutch book argument. The idea behind qualitative probability is that if we could provide qualitative axioms for belief that, when satisfied, were sufficient to yield a total order, then we would have grounds to consider the axioms rational—all without assigning numerical degrees of belief.

Consider a relation $\succeq$ on $\mathcal{F}$, where A, B, C, D are propositions in $\mathcal{F}$, and '$A \succeq B$' is interpreted to say that an ideal agent α deems A to be at least as plausible as B. Any relation $\succeq$ satisfying the following six axioms is a *qualitative probability relation.*

1. If $(A \equiv B)$ and $(C \equiv D)$ are logically true and $(A \succeq C)$, then $(B \succeq D)$;

2. It is not the case that $(A \wedge \neg A) \succeq (A \vee \neg A)$;

3. $B \succeq (A \wedge \neg A)$;

4. $A \succeq B$ or $B \succeq A$;

5. If $A \succeq B$ and $B \succeq C$, then $A \succeq C$;

6. If $\neg(A \wedge C)$ and $\neg(B \wedge C)$ are logically true, then $A \succeq B$ if and only if $(A \vee C) \succeq (B \vee C)$.

Given a qualitative probability relation $\succeq$ with respect to α, we may define an *equivalence relation*, $\simeq$, and also a strict plausibility relation, $\succ$, as follows. First, *equivalence*: $A \succeq B$ and $B \succeq A$ if and only if $A \simeq B$. Next, *strict plausibility*: $A \succ B$ if and only if $A \succeq B$ and not $B \succeq A$. Now let us consider a new axiom, Axiom 7.

7. If $A \succ B$, then, for some n, there are n propositions $S_1, \ldots, S_n$ where for all $S_{1 \leq i \leq j \leq n}$ and $i \neq j$, $\neg(S_i \wedge S_j)$ is logically true , and $(S_1 \vee \ldots \vee S_n)$ is logically true, and such that for each S_i, $A \succ (B \vee S_i)$.

A key result of Leonard Savage's (Savage 1972) is that if α exercises a qualitative probability relation $\succeq$ over $\mathcal{F}$ satisfying these seven axioms, then there is a unique quantitative probability measure such that $\Pr(A) \geq \Pr(B)$ if and only if $A \succeq B$.

Finally, Hawthorne and Bovens's proposal is to add to quantitative probability an axiom for full belief, namely

8. (Hawthorne and Bovens 1999, 262) if $A \succeq B$ (i.e., if α deems A to be at least as plausible as B) and α believes B, then α believes A.

An important point to notice about qualitative probability is that it demands more of ideal agents than just logical omniscience. While transitivity of the strict plausibility relation $\succ$ is uncontroversial, axioms for a strict plausibility relation are not sufficient to yield a total ordering of $\mathcal{F}$—hence Axiom 5, the requirement that *weak* preference $\succeq$ satisfy transitivity. However, Axiom 4 demands that for any two beliefs A, B, a rational agent either finds A more plausible than B, B more plausible than A or will be indifferent between A and B, where indifference amounts to the agent judging each belief of equal epistemic bearing. But judging two beliefs A and B equally plausible is a stronger disposition than having *no* comparative judgment for one *vis a vis* another. It is perfectly consistent for α to be logically omniscient yet not be in a position to either stake one belief more or less plausible than another or judge them to be equally plausible. Indeed, to exclude indecision as a rationally possible state we must state that α is in a context in which all outcomes are comparable (c.f., Hawthorne and Bovens (1999, 246, note 7)),[4] has available to him a qualitative notion of confidence precise enough to put him in a position to satisfy the completeness axiom (Axiom 4) and the cognitive ability to apply this notion to his doxastic states. Logical omniscience alone is insufficient.

This said, notice that qualitative probability doesn't offer an improvement to Hawthorne and Bovens's original account with respect to providing a structured formal system. The crucial concept in modeling their logic of belief within qualitative probability is still α's notion of confidence and this notion must be precise enough for α to effect comparisons that satisfy the completeness axiom. We might pursue a strategy to maintain that confidence is a qualitative notion by introducing quantitative benefits that agents wish to maximize (e.g., money). But notice that this move takes us no closer to articulating a logical calculus for rationally accepted formulas. Hawthorne and Bovens's proposal, whether based directly on the classical probability measure or whether passing through qualitative probability as an intermediary theory, yields the same output: a metalinguistic description of a decision procedure that relies upon a table of α's rational beliefs for us to consult.

7.

To summarize, it was observed that Hawthorne and Bovens's logic of belief is not a logical calculus since it neither includes connectives in the object language for combining rationally accepted sentences nor does it provide logical inference rules that preserve rational acceptability under (restricted) entailment. Rather, what Hawthorne and Bovens do provide is an *unstructured* closure operation extracted from the semantic features of a Lockean conception of belief. It was observed that the rules HB1 and HB2 that the theory generates are sound decision procedures rather than sound rules of logical inference. The reason for this assessment is that the theory provides no formal language capable of expressing formulas that are substitution instances for either HB1 or HB2. So, HB1 and HB2 are necessarily meta-linguistic expressions which are better understood as describing how to calculate consistent rational belief states. For this reason it was concluded that the logic of belief is unstructured.

It was also observed that the notion of qualitative belief does little work in resolving the paradoxes and that there is reason to regard its use to estimate quantitative threshold parameters a handicap. First, the constructive role that full belief plays in the theory is to solve for the threshold parameter q and does not address how to extend the logic to include rationally accepted formulas. Second, even when considering the theory's capabilities for estimating threshold levels, it turns out that more is required to effect estimates than the assumption that α satisfy the Lockean thesis and logical omniscience. In the original theory the class of contexts in which we may precisely estimate the threshold value for q is constrained by the weak lottery context assumption, which is more restrictive than general circumstances involving rationally accepted propositions. The theory of qualitative probability does nothing to relax these constraints but rather adds additional restrictions by requiring that α only be in contexts in which all his beliefs are comparable and that he not be indecisive on pain of failing to satisfy the completeness axiom. With respect to this last point, it was remarked (Note 4) that this feature may present a problem for Hawthorne and Bovens's development of their contextualist interpretation of confidence if a set of propositions varies by being comparable in one context but fails to be comparable in another context. This point presents another type of limitation to applying the logic of belief, but I won't pursue this line here.

In short, a theory purporting to resolve the paradoxes of rational acceptance should address the general conflict between the first and third legislative principles for rational acceptance. The structural view holds that a logical calculus for rational accepted formulas should be a requirement for every formal framework designed to resolve the paradoxes of rational acceptance. The minimum expressive capabilities of an object language should be to express the difference between the probability of a conjunctive (disjunctive) event and the conjunction (disjunction) of probabilistic events. Furthermore, restricted consequence should be defined with respect to a formal language for expressing rationally accepted propositions. A calculus with at least these capabilities would allow us to evaluate the formal features of arguments composed from rationally accepted propositions, giving us a more precise understanding of the general conflict presented by the first and third legislative principles.[5]

Notes

[1] It should be noted that there is disagreement in the literature over whether the lottery paradox and the paradox of the preface both require a solution of the second type. To take one example, John Pollock has argued that the lottery paradox should be resolved by a solution of the first type whereas the paradox of the preface should be resolved by a solution of the second type. Pollock considers the lottery paradox an invitation to commit a mistake in reasoning. He argues that since the lottery paradox is an instance of collective defeat, the correct position should be to deny that it is rational to accept that any ticket of the lottery loses. Pollock regards the paradox of the preface, in turn, to be generated from principles we should accept and, hence, thinks that the paradox of the preface requires a solution of the second form (e.g., (Pollock 1993)). Although I think that both paradoxes call for a substantive (type two) solution and will assume this point in this essay, the general view I advance (i.e., the structural view of rational acceptance) does not turn on how these paradoxes are classified with respect to the appropriate type of solution. What is necessary is a considerably weaker claim, namely that there is at least one paradox generated from reasoning governed by the three displayed legislative principles along the lines that I've described that requires a solution of the second type.

[2] There is a slight deviation in my notation that warrants mentioning. Hawthorne and Bovens mark the distinction between quantitative doxastic states and qualitative doxastic states by discussing two different agents, α and β, who differ precisely with respect to the kind of doxastic states each may entertain: α is an agent whose doxastic states are exclusively quantitative, whereas β is an agent whose doxastic states are exclusively qualitative. In my presentation of their account, I simply use α to denote an ideal agent and then discuss the restrictions we may place on α, including the two distinct types of doxastic notions mentioned above. The main reason for my choosing not to follow their notation is because doing so would obscure a critical point I wish to make. Hawthorne and Bovens hold that there are subtle tensions when moving between quantitative and qualitative notions of belief that the paradoxes of rational acceptance make stand out (1999, p. 241). Indeed, they think that "the *preface* and the *lottery* illuminate complementary facets of the relationship between qualitative and quantitative belief" (1999, p. 244). I reject this analysis, for reasons that will become apparent. The short of it is that I maintain that the apparent tensions between qualitative and quantitative belief that they study are artifacts of their framework that have little to do with either the paradox of the preface or the lottery paradox

[3] There are two points to mention. First, it is important to stress again that the dispute is not over semantics *per se*, but rather the logic of belief's use of semantics in place of a syntax for constructing rationally accepted formulas. For a general discussion of formalized languages and inference rules, see (Church 1944, §07.). Second, it is worth mentioning again the three conditions on a formal language observed in section 2. The objection discussed here is that one cannot begin to evaluate the logic of belief with respect to these conditions because there isn't a formal language for the logic of belief to even construct formal proof objects.

[4] This point may present a problem for Hawthorne and Bovens's contextualism, for they intend their degree of confidence measure to be contextually determined. On their view, an agent who maintains the very same degree of belief across contexts may nevertheless assign different threshold points in different contexts to yield different sets of fully accepted beliefs. For instance, in one context (i.e., by one confidence

measure) an agent may have a common sense belief that a train will arrive on time but fail to believe that two events will occur at the same moment in a controlled experiment, where the different doxastic attitudes is due to a different threshold point for full belief rather than a different degree of confidence assigned to each proposition. Hawthorne and Bovens's remark that their "analysis applies to any single belief standard, and may be applied to each of a number of standards, one by one" (1999, 246, note 7). However, in light of this condition of comparability of belief, notice that this picture of accounting for contextually sensitive thresholds for belief threatens to break down: for qualitative belief in a set of propositions may vary among contexts in the sense that all beliefs are pairwise comparable in one class of contexts but fail to be comparable in another class of contexts. But contexts of the latter type fail to yield a single qualitative probability space, and so a meaningful comparison of thresholds could not be made.

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