

Nature of Mathematical Reasoning: 80-110

Spring 2013

Time & Location:

Monday, Wednesday
11:30 – 12:20 PM
PH A22

Instructor:

Gregory Wheeler
Baker Hall 155C
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Course Website:

<http://www.cmu.edu/blackboard>

Office Hours:

Tuesday 11:00-12:00 or by appointment

Course Description:

If asked to describe the study of mathematics, most people are likely to describe some specific mathematical theory – like calculus or algebra. In fact, the bulk of your mathematical upbringing was probably spent learning to perform different sorts of calculations according to different sorts of rules. But underneath the technical details lies the fascinating world of mathematical reasoning.

In this course, our aim is to gain an understanding of contemporary mathematical reasoning and to see the simple elegance that abstract reasoning can bring to seemingly counter-intuitive problems. We will approach this project with the aid of examples found in the history and philosophy of mathematics, which will help us to better understand how the modern theory of mathematical reasoning has developed. In addition, we will consider several problems that seem to defy any intuitive solution, but which have compelling and in some cases simple solutions when put into mathematical form. These cases will highlight the value of approaching a problem in a carefully considered, mathematical manner.

Course Objectives:

Upon successful completion of this course you should be able to:

- place developments in mathematics into their historical contexts, and explain how these developments have impacted the evolution of mathematical reasoning;
- identify and give examples of fundamental proof strategies in mathematics;
- explain and give examples of axiomatic mathematical theories;
- explain the sense in which formal mathematics is and is not objective and rigorous;
- provide rigorous mathematical solutions to several famous exam ples

Readings:

The textbook for this course is *More Precisely: The Math You Need to Do Philosophy*, by Eric Steinhart. Additional readings and resources will be accessible via Blackboard. Several readings will come from *The Language of First-Order Logic*, by John Barwise & John Etchemendy, which is out of print but available used in many places.

Grading:

Item	Portion of Grade
Homework Assignments (8-10)	30%
Midterm Exam	30%
Final Exam	30%
Class Participation	10%
Final Paper (optional)	5%

Homework Assignments (30%):

Assignments will be handed out in class on Wednesday, and will be due by the beginning of class the following Wednesday. Some assignments will ask you to use skills or concepts learned in class and to exercise your philosophical and formal reasoning muscles. Others will ask you to go outside of the readings and materials from class and find new examples, materials, etc.. The types of questions posed on homework sets will vary, but will typically involve doing some reading and writing, and solving introductory level formal problems. Homework problems that involve solving formal problems may be done in groups – but the write-up should be your own. Homework assignments can be accessed via Blackboard. A hardcopy of your assignment should be turned in the beginning of class on its due date. Your assignment should be typed and should include your name, Andrew ID, and the Assignment number.

Homework Grading Scheme:

- 5 = excellent
- 4 = good
- 3 = nice try
- 0 = no try or late.

Exams (30%):

Exams will cover material from classes up to the date of the exam. The final exam will be comprehensive. A sample exam will be provided a week before the actual exam, and reviewed in class.

Exam Grading Scheme:

- 100 point scale

Final Paper (5%):

An optional 5-10 page paper on a topic related to the course material will be due at the last class. The paper can be on a topic of your choice, but must be approved by the instructor prior to Thanksgiving. More details will be distributed in class (and posted on Blackboard).

Final Paper Grading Scheme:

- 5 = excellent
- 4 = good
- 3 = nice try
- 0 = no try or late.

Policies:

Attendance: Attendance is not required. However, 10% of your grade is class participation, and it is impossible to participate without attending. If you miss a class it is *your* responsibility to find out about the content of the class you missed. You are also responsible to meet deadlines for assignments that are due on the day you are absent, or make other arrangements.

Late Assignments:

- Late homework assignments will be given a 25% penalty per day. Homework assignments are due at noon on Thursdays.

Also note that all students will be expected to take the midterm and final exam at the same time as other students in the class (except if there is a conflict, as per CMU's policy on final examinations, available here: <http://www.cmu.edu/policies/documents/Exams.htm>). If you foresee a problem, please speak with me about it as soon as possible.

Laptops, Cell Phones and Related Technology: I would prefer no laptop use (they are distracting), but if you prefer taking notes on the computer, feel free. Please turn your cell phones off during class.

Plagiarism & Cheating: Collaboration on homework assignments is permitted, but student should make an effort to write up their own solutions. Plagiarism and cheating are serious academic offences with serious consequences. If you are discovered engaging in either behaviour, you will receive a failing grade on the assignment or exam in question, and further disciplinary action may be taken, in accord with the university's policies.

For information on what constitutes plagiarism and cheating, and what the university policies are, please check out the following link:

<http://www.cmu.edu/policies/documents/Cheating.html>

Topics

1. The History of Mathematics
2. The Nature of Mathematical Arguments/Proofs
 - a. Reductio Ad Absurdum (Indirect Proof)
 - b. Arguments by Cases
 - c. Mathematical Induction
3. Axiomatic Theories
 - a. Axioms
 - b. Definitions
 - c. Examples
 - i. Euclidean Geometry
 - ii. Peano Arithmetic
 - iii. Causation
 - iv. Others – Rational Choice, Probability, Etc.
4. Formalizing Mathematical Reasoning
 - a. Formal Logic
 - b. Formal Proofs
 - c. Proofs by Computer
5. Applications
 - a. Probability Theory
 - i. Lets Make a Deal
 - ii. Hume on Miracles
 - iii. Medical Tests
 - b. Infinity
 - i. The Size of Infinite Sets

Date	Topic	Reading	Notes
Jan. 14	Course Introduction		
Jan. 16	Origins and History of Mathematics	Ch 1 More Precisely, Sec 1, 2, 3	
Jan. 18	Part I: The Nature of Arguments & Proofs	Ch 1 More Precisely, Sec 4, 5	
Jan. 21	No Class (MLK day)		
Jan. 23		Ch 1 More Precisely, Sec 6, 7	Homework 1 due
Jan. 25		Ch 1 More Precisely, Sec 8, 9	
Jan. 28		Ch 1 More Precisely, Sec 10, 11	
Jan. 30		Ch 1 More Precisely, Sec 12, 13	Homework 2 due
Feb. 01		Ch 1 More Precisely, Sec 14, 15,	
Feb. 04	No Class	Ch 1 More Precisely, Sec 16, 17	
Feb. 06	Part II: The Nature of Axioms	Ch 1 More Precisely, Sec 18	Homework 3 due
Feb. 08		Ch 1 More Precisely, Sec 19	
Feb. 11		Ch 2 More Precisely, Sec 1, 2	
Feb. 13		Ch 2 More Precisely, Sec 3, 4	
Feb. 15		Ch 2 More Precisely, Sec 5,	
Feb. 18		Ch 2 More Precisely, Sec 6	
Feb. 20		Ch 2 More Precisely, Sec 6	Homework 4 due
Feb. 22	Part III: The Nature of Numbers	Ch 2 More Precisely, Sec 7	
Feb. 25		Ch 2 More Precisely, Sec 8, 9	
Feb. 27		Ch 2 More Precisely, Sec 10	Homework 5 due
Mar. 01		Ch 2 More Precisely, Sec 11	
Mar. 04			
Mar. 06	Midterm Exam		
Mar. 08	No Class (Mid-Semester Break)		
Mar. 11	No Class (Spring Break)		
Mar. 13	No Class (Spring Break)		
Mar. 15	No Class (Spring Break)		
Mar. 18	Part IV: The Nature of Formal Reasoning	Ch 2 More Precisely, Sec 12	
Mar. 20		Ch 2 More Precisely, Sec 13	
Mar. 22		Ch 2 More Precisely, Sec 14	
Mar. 25		Ch 2 More Precisely, Sec 15	
Mar. 27		Ch 2 More Precisely, Sec 16	Homework 6 due
Mar. 29			
Apr. 01			
Apr. 03	Part V: The Nature of Reasoning Under Uncer	Ch 5 <i>More Precisely</i> Sec 1, 2, 3	HW 7 & Paper Outline due
Apr. 05		Ch 5 <i>More Precisely</i> Sec 4, 5, 6	
Apr. 08		Ch 5 <i>More Precisely</i> Sec 7, 8	
Apr. 10		Ch 6 <i>More Precisely</i>	Homework 8 due
Apr. 12		Ch 6 <i>More Precisely</i>	
Apr. 15	Part VI: The Nature of the Infinite	Ch 2 <i>More Precisely</i> Sec 17, 18	
Apr. 17		Ch 7 <i>More Precisely</i> Sec 1	Homework 9 due
Apr. 19	No Class (Carnival)		
Apr. 22		Ch 7 More Precisely Sec 1	
Apr. 24		Ch 7 More Precisely Sec 2	
Apr. 26		Ch 7 More Precisely Sec 3	Homework 10 due
Apr. 29		Ch 7 More Precisely Sec 4	
May. 01		Ch 7 More Precisely Sec 5	
May. 03	The Nature of Mathematical Reasoning		Final Paper due